# Instituto Superior de Economia e Gestão <br> Operational Research - $1^{\text {ST }}$ Semester 

## Date: 12/01/2016

(Note: Justify all your answers and present all the calculations)

1. A company aims to maximize the weekly profit with the production and sales of products $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$.
To produce A,B and C, 240 hours machine (h.m) are available per week. Prior to selling, products need to be in an oven that has $700 \mathrm{~m}^{3}$ capacity. The company cannot sell more units of $\mathbf{B}$ and $\mathbf{C}$ together than $\mathbf{A}$. An agreement forces the production of a minimum of 20 units of $\mathbf{C}$ every week. The corresponding LP formulation follows:

$$
\begin{aligned}
& \max Z=10 x_{A}+20 x_{B}+40 x_{C} \\
& \text { s.t. }: \\
& \begin{array}{c}
x_{A}+x_{B}+4 x_{C} \leq 2400 \\
4 x_{A}+2 x_{B}+x_{C} \leq 700 \\
x_{A}-x_{B}-x_{C} \geq 0 \\
x_{C} \geq 20 \\
x_{A}, x_{B}, x_{C} \geq 0
\end{array}
\end{aligned}
$$

By Solver/Excel the following "Sensitivity Report" was obtained:

## Microsoft Excel 15.13 Sensitivity Report

Variable Cells

| Cell | Name | Final <br> Value | Reduced <br> Cost | Objective <br> Coefficient | Allowable <br> Increase | Allowable <br> Decrease |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\$ \mathrm{~B} \$ 7$ | $x_{A}$ | 90 | 0 | 10 | 10 | 16,6667 |
| $\$ \mathrm{C} \$ 7$ | $x_{B}$ | 70 | 0 | 20 | $1 \mathrm{E}+30$ | 10 |
| $\$ \mathrm{D} \$ 7$ | $x_{C}$ | 20 | 0 | 40 | 25 | $1 \mathrm{E}+30$ |

## Constraints

| Cell | Name | Final <br> Value | Shadow <br> Price | Constraint <br> R.H. Side | Allowable <br> Increase | Allowable <br> Decrease |
| :---: | :--- | :---: | :---: | :---: | :---: | :---: |
| $\$ \mathrm{E} \$ 2$ | machine (h.m.) | 240 | 15 | 240 | 60 | 140 |
| $\$ \mathrm{E} \$ 3$ | oven $\left(\mathrm{m}^{3}\right)$ | 520 | 0 | 700 | $1 \mathrm{E}+30$ | 180 |
| $\$ \mathrm{E} \$ 4$ | sales - A vs B \& C | 0 | -5 | 0 | 140 | 180 |
| $\$ \mathrm{E} \$ 5$ | sales of C | 20 | -25 | 20 | 28 | 18 |

Answer all the following questions based only on the "Sensitivity Report" and consider them independent.
a) (1,5 points) News about harm to health caused by $\mathbf{C}$ led to a $10 \%$ decrease in the profit of that product. Evaluate the consequences of such a change.
b) (2,0 points) An explosion in the factory results on a decrease in 30 hours machine per week in the capacity. Determine the weekly profit of the company after the accident, justify if there is a change considering the initial profit.
c) ( 1,0 point) Justify if it is possible to make available for other products a part of the capacity of the oven, maintaining the production plan given in the report
2. ( 1,0 point) Consider the following network, where the values on the edges represent time. Justify without solving what will be the effect in the optimal value of the minimal spanning tree in this network if Prim algorithm starts by vertex $\mathbf{B}$ instead of vertex $\mathbf{A}$.

3. The weekly distribution of waters for three big supermarkets ( $\mathbf{S 1}, \mathbf{S 2}$ and $\mathbf{S 3}$ ), could be done from two warehouses (A1 and A2). The warehouses have a weekly capacity of 1000 and 800 boxes, respectively. The unit transportation cost in m.u. (monetary units) and the number of boxes requested by each supermarket are in the table below.

|  | S1 | S2 | S3 | Capacity <br> (no. boxes/week) |
| :---: | :---: | :---: | :---: | :---: |
| A1 | 5 | 2 | 9 | 1000 |
| A2 | 2 | 4 | 5 | 800 |
| Weekly demand <br> (no. boxes) | 900 | 300 | 400 |  |

a) (2,0 points) Formulate the problem using an LP model.
b) $(0,5$ points) Justify if there is need to change the model to ensure that the optimal solution has integer values, if it is the case indicate the changes needed.
c) (2,0 points) Assume that the company has to decide if it should open a new warehouse (A3) with capacity for 2000 boxes that will work only with the three supermarkets. If the option is to open A3, warehouses A1 and A2 could be used to other purposes. The construction cost is $5000 \mathrm{~m} . \mathrm{u}$., and the unit transportation costs are equal to all the supermarkets, and equal to $0.5 \mathrm{~m} . \mathrm{u}$. Present one model that helps to find the best option for the new problem (without need to solve the problem formulated in a) alone ).

